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論文題目：

Portfolio selection and trading by using multi-objective Genetic Algorithm

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Abstract

The well-known mean-variance model cannot satisfy investors' request for different investment preference and risk diversification. Consequently, we consider genetic algorithms for portfolio selections which consider risk preference including return, risk, liquidity, return distribution and transaction cost. Further, we try to improve the Markowitz model by multi-objective genetic algorithms (MOGAs). Why we used MOGAs? Because of MOGAs have considered all the objectives in the same time with solving quadric programming problem and optimized the solution in globally pareto optimal. Moreover, Multiobjective functions are prior than single objective because of solving the conflicts exquisitely in complex objections. Multiobjective genetic algorithms (MOGAs) can explain the trade-off between return and risk which behavior finance investigates. This paper proposed method which incorporate different risk measures, skewness, entropy, liquidity and transaction cost. A trading example is also illustrated to compare with the proposed method. On the basis of the numerical results, the method we proposed can provide a higher return on asset and having better risk diversifications.

Keywords: multi-objective genetic algorithms (MOGAs); portfolio selection

Ⅰ. General Background

The purpose of this study is to develop a model for portfolio optimization by using multi-objective genetic algorithm (MOGA). There are some improvements for the expanded model in portfolio selection and optimal portfolio construction, such as using entropy to measure the divergences avoiding the most criticisms in Markowitz's mean-variance model which often extremely concentrated on a few assets and difficultly solving a large-scale quadratic programming problem and using covariance to represent the risk of interrelations under uncertainty, helping investors to control or hedge the risk in portfolio.

Portfolio theory which is initially expounded by Markowitz (H. Markowitz, 1952; H. M. Markowitz, 1959), even though the proposed model turns into the base of portfolio theory, obviously it has some problems needed to improve. For instance, both of us want to find the portfolio with minimization of mean-risk and non-convex transaction cost, how to define the optimal transaction lot and the number of asset in portfolio, not often extremely concentrated on a few assets, difficulty solving a large-scale quadratic programming problem. Therefore, some improvements use genetic algorithms for handing preceding problems (J. S. Chen, Hou, Wu, & Chang Chien, 2009; C. C. Lin & Liu, 2008; Soleimani, Golmakani, & Salimi, 2009; Wilding, 2003).

Recently, multi-objective genetic algorithm (MOGA) is important and widely used in chemical engineering since it can have significant impacts on making the best choices in pareto optimal solutions (Dietz, Azzaro Pantel, Pibouleau, & Domenech, 2008; Marseguerra, Zio, & Podofillini, 2004; Osman, Abo Sinna, & Mousa, 2005a). Statistical methods and data mining technique have been used for finding the best portfolio within maximize the return and minimize the risks which the risks are company risk and market risk (Chang, Yang, & Chang, 2009; P. C. Lin & Ko, 2009). In the past, when we encounter most of the tri-objective programming model problem, the most common way is making the bi-objective model to a single objective (J. S. Chen, et al., 2009; Wilding, 2003). However, the previous way can't outperform optimal solutions because of it does not thinks all of objectives. Therefore, we need to use multi-objective genetic algorithm, it is much powerful than just thinking bi-objective algorithm (such like Markowitz's mean-variance model) that assists us to construct a portfolio under the minimum or acceptable risk.

Moreover, all the thing investors interested is portfolio risk and how to separate the risk or customized according investors' preference. By this reason, this paper uses different risk measure methods such as covariance and semivariance for different investors' preferences. In addition, we take liquidity into account which in securities markets is a rapidly growing issue in both of the investors and financial researchers. Changing with time, liquidity would follow the market fluctuations, crises or bloom some kinds of real economic stress. For this reason, recently when talk about liquidity we would investigate the consequences of systematic fluctuations in liquidity.

Ⅱ. Literature Review

Portfolio theory and transaction cost

In mean-variance model the problem is an optimization problem involving two criteria: first, the mean should be maximizes and the risk must be minimized. Hence, the performance of the mean-variance approach depends on the accurate forecast the return rate or risk (which can't be obtained the accurate number, it just guesses). Therefore, some research (Z. Chen & Wang, 2008; Daníelsson, Jorgensen, Vries, & Yang, 2008; Hasuike, Katagiri, & Ishii, 2009; K. Y. Huang & Jane, 2009; Soleimani, et al., 2009) used different method for measured the risk (expected return, uncertainty risk and relation risk) in portfolio selection. That is to say, risk measure in portfolio selection is the most important things not which methods we used.

Many results construct on the assumption of no transaction costs which initial investment required is linear function of the price. However, some research (Arnott & Wagner, 1990; Chellathurai & Draviam, 2007; Y. Fang, Lai, & Wang, 2006; Pelsser & Vorst, 1996) show the significant of transaction cost without transaction cost will bring no efficient portfolio selection and it is necessary to control transaction cost in optimal situation that can make investor or researcher have a better constrain of constructing portfolio with better improved investment performance.

The other drawbacks of Markowitz mode are time consuming for computational difficulty in solving a large-scale quadratic programming problem and can't consider all objectives in the same time. Therefore, some scholars introduce evolution algorithms for handling this issue (Chang, et al., 2009; J. S. Chen, et al., 2009; C. C. Lin & Liu, 2008). Moreover, for the purpose of considering all objective in the same time we also use multi-objective evolutionary algorithms based on genetic algorithms which calls multi-objective genetic algorithms (MOGA) for including transaction cost, return, risk, liquidity and entropy.

Genetic algorithm (GA) in portfolio

Genetic algorithm (GA) is a random searching optimization tool combined with mathematics and biology which can iterate the process to find the suitable generation by mutation and crossover. Genetic algorithm (GA) is a powerful and random optimized selection method, imitating the nature evolutionary and survival of fitness, through competition and reserving suitable individual to gaining the optimal solution for constructing portfolios (J. S. Chen, et al., 2009; Wilding, 2003).

Recently, some researches using genetic algorithm (GA), to improving portfolio selection and transaction problem. Portfolio selection optimal problems were hardly computation with quadratic or non-linear programming models (C. C. Lin & Liu, 2008; Soleimani, et al., 2009).

Furthermore, the proposed method can obtain nearly optimal and possible implemented solution within a short period closing to the efficient frontier. For coping with the classical portfolio problem of distributing capital to a set of securities in investment strategy offer aggressive strategies for effective ways(J. S. Chen, et al., 2009; J. S. Chen & Lin, 2009). Some use GA to find portfolio optimization in measuring different level of risk through Markowitz model for efficient set and volatility forecasting (Chang, et al., 2009; P. C. Lin & Ko, 2009).

In this study we can easily find the accuracy rate having some improvements than previous methods that are better in stock market forecasting. Furthermore, the multi-period can be the better way to realize the input factor of variable because training days is more than forecasting day which can be a better result of forecast according to stock market dynamic hardly to predict.

Multi-objective Genetic algorithm (MOGA)

A multi-objective GA (MOGA) is proposed to solve multi-objective problems integrated with continuous and discontinuous variables. Multi-objective problems are heavily discover in chemical engineering, ranging from applications of process design to determination of optimal operating conditions, and involve simultaneous optimization of several incommensurable and often competing objectives (Deb, 2001; Dedieu, Pibouleau, Azzaro Pantel, & Domenech, 2003; Dietz, et al., 2008; Marseguerra, et al., 2004).

For instance, if we want to design a process, we want to normalize the formulation which minimize the investment cost or maximize the investment profits, but at the same time, we want to minimize the environmental efforts (stock market flexibility or each of the stock covariance). There are some advantages compared with single-objective optimization because it has some trade-off when consider all of constrains and the optimal solutions which is famous for Pareto-optimal solution (Deb, 2001; Goel, et al., 2007; Suga, Kato, & Hiyama, 2009).

What is the definition of Pareto optimal? Pareto efficient situations are those in which any (additional) change to make any person better off is impossible without making someone else worse off. Pareto optimal set (or Pareto optimal subset), representing the best possible objective values, is the optimal solution that we can use to construct a surface according our interested objective. Moreover, in practice, we may not always be interested in finding the best solutions but investors' preference (different objectives mean different customer spectrum).

Most of the multi-objective optimization problems have many optimal solutions which are global optimal compared than single objective. Anyone of the optimal solution demonstrates different compromise among the objectives which we design or the restrictions should be considered (Abido, 2003; Gutiérrez-Antonio & Briones-Ramírez, 2009). Hence, Goel (2007) is interested in finding as many as possible Pareto optimal solutions for the sake of selecting a compromise that agree with investors' preference. These solutions are no-more better than others in the search base when all other solution are considered. Dietz (2008) mentioned multi-objective genetic algorithm (MOGA) was able to finding the optimal solutions for each objective or constrains, which be regarded as Pareto optimal solution, and MOGA exactly can handle the various problems with flexibility and adaptability. MOGA can be used in safety system to optimize the solutions satisfying the other target and requirements, for an efficient search through the solution space using a multi-objective genetic algorithm which allowing to identify a set of Pareto optimal solutions providing the decision maker with the complete spectrum of optimal solutions with respect to the various targets, and the decision maker can select the best solutions conforming with the objectives (Busacca, Marseguerra, & Zio, 2001; Marseguerra, et al., 2004).

Why we should use multi-objective genetic algorithms? Because MOGA can considers many restrictions in the same time better than single objective in traditional linear searching space which cannot find better solutions if the searching space is non-linear. Furthermore, in the real world or the target function which we want to know, most of the optimization problems are multi-objective (Marseguerra, et al., 2004; Osman, Abo Sinna, & Mousa, 2005b; Suga, et al., 2009), where the restrictions (objectives) that should be taken into account simultaneously and independently, and those are helpful for managers to analyze the results that approximate the investors' preference.

Moreover, there are some investigations using genetic algorithms in solving the multi-objective resource allocation problems (MORAPs) with significant results. Those studies (Osman, et al., 2005a, 2005b) using genetic algorithm for dynamic programming (DP) to improve the computation efficient of select mathematical programming problem. Chen, Mcphee and Yeh (2007) points out that its simulation results with better solutions and closer to the true Pareto frontier which provides viable alternative to solve the problem to optimization. Ripon, Kwong and Man (2007) using a modified multi-objective genetic algorithm indicate an optimal solution with Pareto-optimal front and better address the issues regarding convergence and diversity in multi-objective optimization.

In addition, we take liquidity into account as well. Liquidity risk in securities markets is a rapidly growing issue in both of the investors and financial researchers. Furthermore, there are some interesting papers about liquidity (Gatev & Strahan, 2009; J. Huang & Wang, 2010; Johnson, 2006). The final goal is using newly mathematic algorithms (multi-objective genetic algorithms) to achieve an objective how to allocate the capital finding the suitable portfolio and maximizing the investor’s profits to increasing the most people welfare.

Ⅲ.Method

Research design

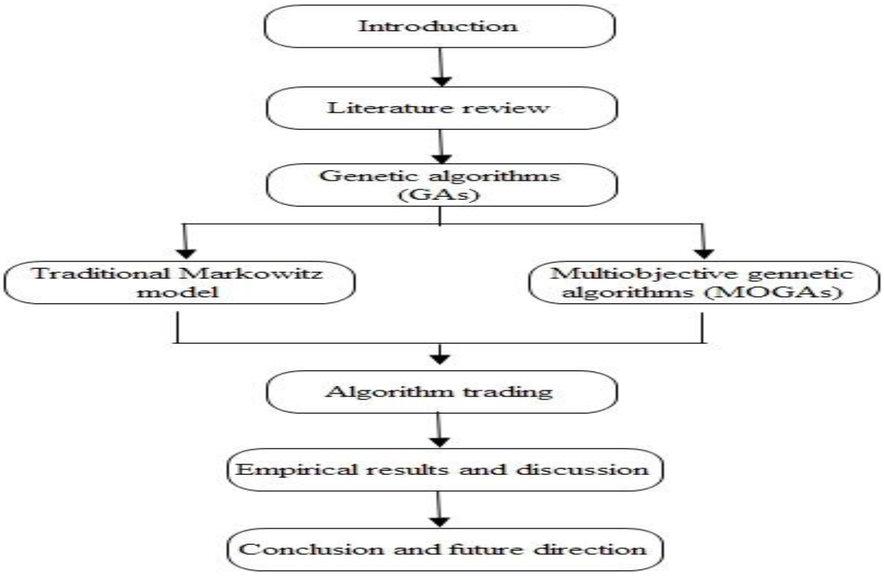

Fig. 1 Steps for research process

Genetic algorithm

Genetic algorithm was beginning from Holland in the early 1970s and particularly his book Adaptation in Natural and Artificial Systems (1975). Genetic algorithm comes from Darwin’s theory, survival of fitness and natural selection which is an evolutionary algorithm that use techniques inspired by evolutionary biology such as selection, crossover and mutation, and used as simple models of evolutionary processes for solving the optimization problem.

Entropy

Entropy has provided us with an estimate of information content and we may also want to compress it to the theory of lower bounds. We use entropy to measure the divergence of portfolio(S. C. Fang, Rajasekera, & Tsao, 1997; Kapur, 1990).

Skewness

Two kinds of skewness a distribution:

1. Negative skew: The left tail is longer; the mass of the distribution is concentrated on the right of the figure. It has relatively few low values. The distribution is said to be left-skewed.
2. Positive skew: The right tail is longer; the *mass* of the distribution is concentrated on the left of the figure. It has relatively few high values. The distribution is said to be right-skewed.

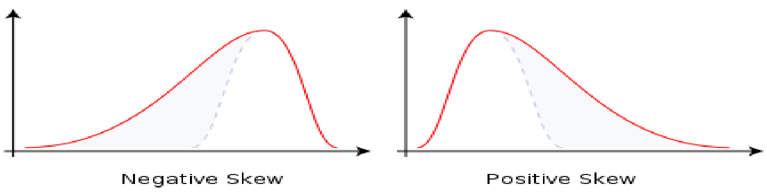

Fig. 2. Skewness patterns

Local pareto optimal vs. Globle pareto optimal

how to separate the final solutions to local pareto optimal or globlly pareto optimal solution is the most important thing. The following figures show how we make sure that final solution is local optimal or globle optimal (Deb, 2001).

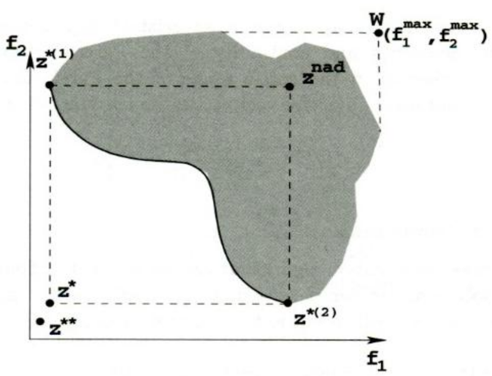

Fig.3 The ideal solution sets

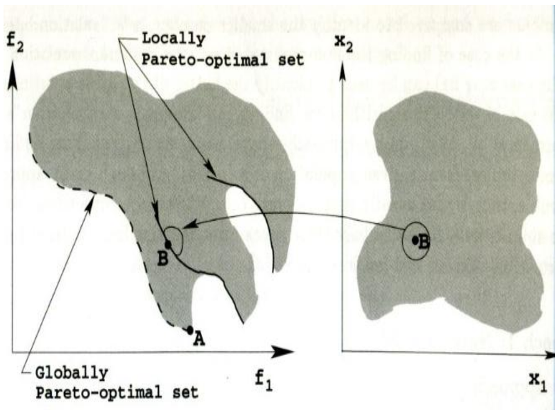

Fig.4 Globally optimal vs. Local optimal

How can we make sure that our finding is the suitable solutions? According to figure 3, the brown area is all of the possible solution set. But how can we pick up the most useful solution set to construct the pareto optimal line is the thing that we eager to know. f1 and f2 are two objectives which can make an solution set and from this solution set can obtain pareto optimal line(from $z^{*(1)} to\_z^{*(2)}$) which line is under the most suitable trade-off situation.

Data Collection Procedure

The data, the data set is used in this study from Morgan Stanley Capital International (MSCI) and random selections from MSCI index. The samples we choose from MSCI Taiwan index are 165companies.

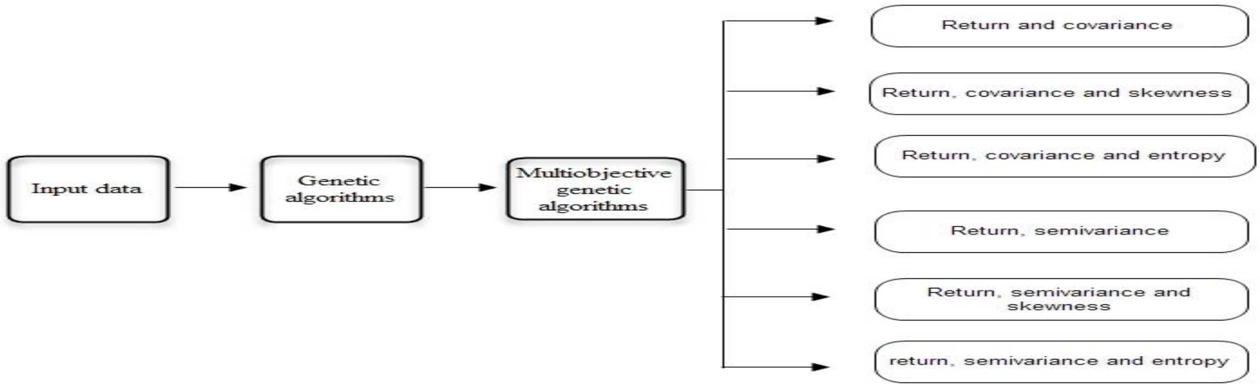

Fig. 5. MOGAs internal process

Ⅳ Empirical results

The difference between ten, fifteen and twenty firms

We follow those thinking steps in our study which used the medium value for deeply research. The next steps we compare 10, 15 and 20 firms with different skewness level, covariance, semi-variance (alfa 0.1 and 0.3) and entropy.

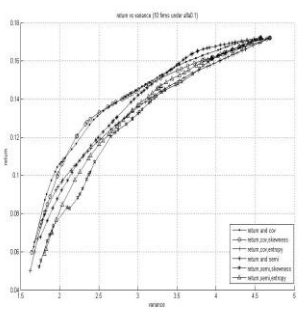

Fig. 6a. Return and variance(10 firms under alfa0.1)

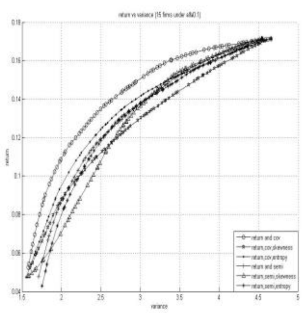

Fig. 7a. Return and variance(15 firms under alfa0.1)

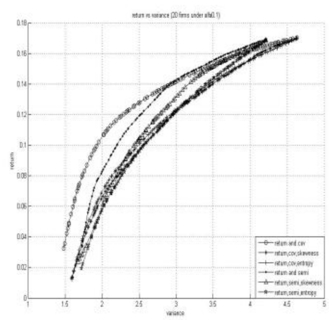

Fig. 8a. Return and variance (20 firms under alfa0.1)

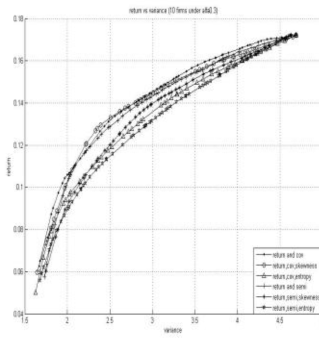

Fig.6b.Return and variance(10 firms under alfa0.3)

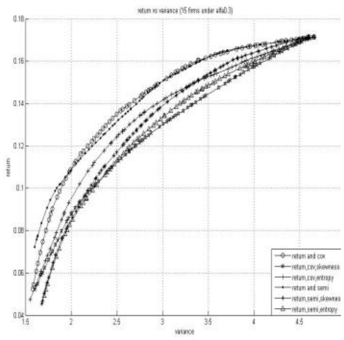

Fig. 7b. Return and variance (15 firms under alfa0.3)

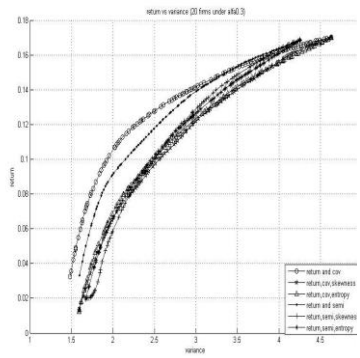

Fig. 8b. Return and variance (20 firms under alfa0.3)

From Figure 8a to 8b are ten firms with different investors' preference and we used two type of measured variance to find different variances with different portfolios which process different characteristics.

We can see the line with two objectives (return and covariance or return and semivariance) is more profits because of a few restrictions. Further, in ten firms return, covariance and skewness are suitable for investors to judge whether invest or not. Because of its achievement closed to barely two objectives. Moreover, objective with entropy seems with poor results because it's a customized behavior (entropy would make investment not focus on a few assets).

In this study, we focus on negative position skewness which presents higher risk and higher return. All we want to do are providing different objectives with different return and risk according investors' preference. Risk attitude is people's attitude to risk and we present the different type of risk preference in different risk attitude. For instance, from Fig.9 the risk aversion may choose three objectives (return, semivariance and entropy) and the risk lover may choose two objectives (return and covariance) according to their own preferences.

From Figure 7a to 8b we find that with more objectives (or restrictions) will bring worst solutions but it corresponds to investors' requests. Moreover, under our skewness assumptions semivariance (conservation index) is powerful than covariance so we can find the line with semivariance is lower than covariance.

In sum, with different objectives correspond to different customized preferences. From upper figures, there are some common characteristics which with lesser objectives would bring more profits. However, when we add more objectives in our restrictions it would bring lesser profits. It is obviously that more restrictions make the searching space much smaller in a possible solution space. Therefore, different objectives mean different customer spectrum. Hence, our work is trying to maximize the investors' utility for separating the variance. Therefore, we offer some methods for investors to optimize their portfolio which are suitable for their preferences.

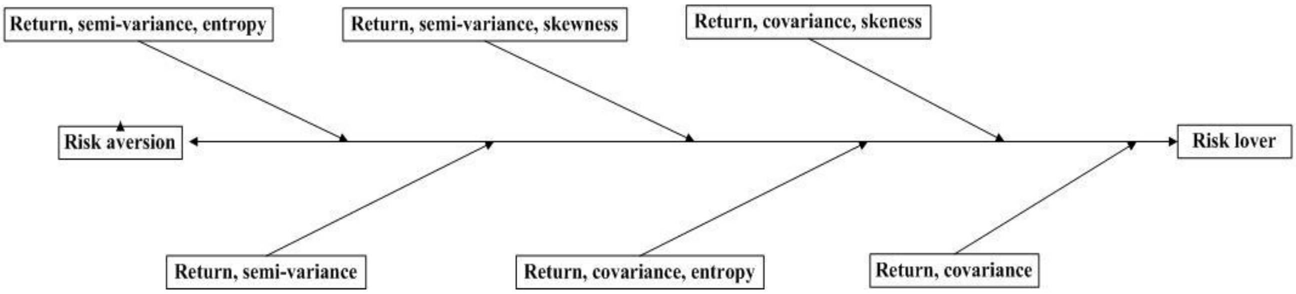

Fig. 9 Type of risk preferences

Observed the difference between two, three objectives

In this section, we compare in different portfolios with two or three objectives. The following figures point out investors can follow up his personal preference for different degree of return (variance) which they pursuits. In our study, we show up ten, fifteen and twenty firms with two objectives (return covariance; return semivariance) or three objectives (return, covariance and skewness; return, covariance and entropy; return, semivariance and skewness; return, semivariance and entropy).

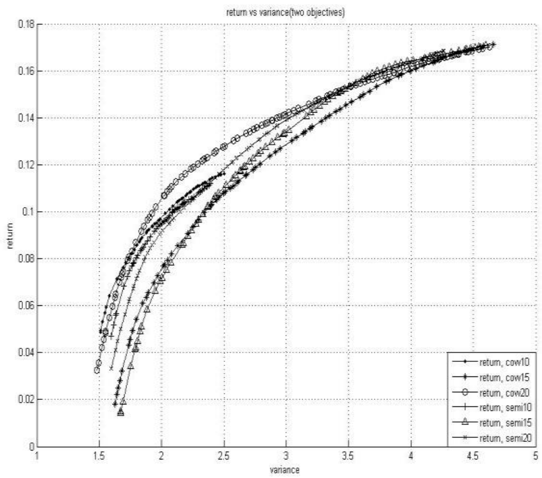

Fig. 10a. Return and variance (two objectives)

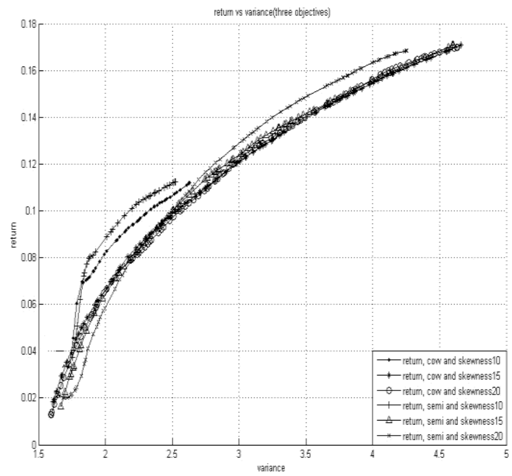

Fig. 10b. Return and variance (three objectives)

The more firms in portfolio would bring the larger searching space so there are more solutions. In Fig. 10a, we can see the most profitable is portfolio which composed of twenty firms with two objectives and the next is fifteen firms with two objectives. There are some similarities, higher variance accompany with higher return. And ten firms with two objectives have lower return and risk because of candidate can be selected in portfolio.

It is obviously that considering more firms the solution space would be larger so that the return of twenty firms would be higher. Moreover, in Fig.10a we wanted to compare two methods of measured variance (covariance and semivariance) which they are famous for value at risk (VaR). From preceding figures, we can see more candidates (larger searching space) would obviously bring the differences between covariance and semivariance, but lesser candidates with smaller differences (smaller firms with smaller searching space.)

In fig.10b we add skewness into our objective for comparison. There is something fun for twenty firms with semivariance, in higher risk level would bring higher return but in lower risk level would bring lower return. Further, twenty firms with covariance and fifteen firms almost have the same results. Moreover, ten firms have the same movement closely.

Furthermore, we add entropy to our objective for compared with skewness and the results in entropy have the similar patterns. Moreover, the points of all efficient frontier line in our study we have listed in appendix Ⅰ.

In sum up, the pattern of solutions is not covergent so even using single objective or aggregative weights the results are worse. Furthermore, the more candidates mean the larger searching space and higher profits. There are the similar patterns in different objectives. Therefore, we offer a novel method for customized portfolio selection by their risk preferences.

Trading comparability

In this section, we compared the trading profits under different fitness function and choosing the suitable fitness value as a foundation for later research development.

Table 1. Trading comparability in different objectives and firms

10 firms	cov			alfa0.3			alfa0.1			alfa0.5		
	return cov	return cov skewness	return cov entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy
Min	0.4382	0.5299	0.4259	0.4177	0.4404	0.4417	0.4641	0.5012	0.4521	0.4089	0.4172	0.4335
max	0.9653	0.9653	0.9652	0.9645	0.9651	0.9650	0.965	0.9654	0.9652	0.9646	0.9652	0.9647
mean	0.7560	0.7636	0.7673	0.7339	0.7728	0.7673	0.7571	0.7744	0.7746	0.7376	0.769	0.7833

15 firms	cov			alfa0.3			alfa0.1			alfa0.5		
	return cov	return cov skewness	return cov entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy
Min	0.5922	0.5967	0.5911	0.6182	0.5388	0.5947	0.5282	0.565	0.5789	0.5634	0.5671	0.5953
max	0.9641	0.9644	0.965	0.9641	0.965	0.9645	0.9641	0.9645	0.9649	0.964	0.9645	0.9644
mean	0.7384	0.8216	0.8172	0.7794	0.7812	0.8343	0.7755	0.8555	0.8034	0.7918	0.8156	0.7914
20firms	cov			alfa0.3			alfa0.1			alfa0.5		
	return cov	return cov skewness	return cov entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy	return semi	return semi skewness	return semi entropy
min	0.5613	0.7847	0.7837	0.7278	0.6634	0.7588	0.7083	0.6739	0.7009	0.7138	0.7402	0.7591
Max	0.9665	0.9669	0.9667	0.9249	0.9261	0.9260	0.9261	0.9256	0.926	0.9261	0.9262	0.9259
Mean	0.7567	0.8828	0.8740	0.8248	0.8230	0.8514	0.8052	0.7987	0.8575	0.799	0.815	0.8584

From the preceding table, the improvements are easily observed under different objectives and firms. General speaking, the function with more objectives would conform with investors' preference even not the best solutions. By this reason, we have some improvements in our research such under the same objectives but we have higher return or approximately return. Furthermore, we compared the general trading and our method that we make the return more stable not change so volatile and more profitable (general trading is 77% but in our method is from 70% to 85%).

Ⅴ. Conclusions

Multiobjective functions are prior than single objective because of solving the conflicts exquisitely in complex objections. Multiobjective genetic algorithms (MOGAs) can explain the trade-off between return and risk which behavior finance investigates (Doran, Peterson, & Wright, 2010; Frankfurter & McGoun, 2002; Rossi, Schwaiger, & Winkler, 2009; Shefrin, 2006).

In this paper a tri-objective portfolio optimization problem with selection and evaluation has been proposed to deal with multi-objective models. The first two objectives were the variance (covariance and semivariance) and expected return as popular using in portfolio selection problems, and the third objectives (skewness and entropy) measure the different investors' preference that how many units they should invest in the portfolio or how the divergence we can accept in the portfolio.

We have combined with covariance, semivariance, skewness, liquidity, transaction cost and entropy for finding an approximation of fantastic trade-off between return, risk and investors' preference of the portfolio. Furthermore, we also have visual comparisons that different firms between covariance and semivariance or same firms with combined objectives (add skewness or entropy) for generating surfaces with optimal arrangement in general. There are some return improvements in computation comparisons for proving our novel method with higher profitability and lower risk level under customer' preference.

On the basis of the numerical results, the method we proposed can provide a higher return on asset and having a better risk diversification results. From the preceding numerical trading table, we have higher return in our portfolio than using simple Markowitz model based on genetic algorithms such as higher return in more objectives or approximately return with more objectives.

Future research can use the data in other countries compared with Taiwan's and lager the composition of portfolio. Further, the later researcher can use the other method measures risk level for better evaluating the risk finding the better solutions.

Ⅵ Reference

Abido, M. A. (2003). A niched Pareto genetic algorithm for multiobjective environmental/economic dispatch. *International Journal of Electrical Power & Energy Systems, 25*(2), 97-105.

Arnott, R. D., & Wagner, W. H. (1990). The Measurement and Control of Trading Costs. *Financial Analysts Journal, 46*(6), 73.

Bingul, Z. (2007). Adaptive genetic algorithms applied to dynamic multiobjective problems. *Applied Soft Computing, 7*(3), 791-799.

Busacca, P. G., Marseguerra, M., & Zio, E. (2001). Multiobjective optimization by genetic algorithms: application to safety systems. *Reliability Engineering & System Safety, 72*(1), 59-74.

Chang, T. J., Yang, S. C., & Chang, K. J. (2009). Portfolio optimization problems in different risk measures using genetic algorithm. *Expert Systems with*

Applications, 36(7), 10529-10537.

Chellathurai, T., & Draviam, T. (2007). Dynamic portfolio selection with fixed and/or proportional transaction costs using non-singular stochastic optimal control theory. *Journal of Economic Dynamics and Control, 31*(7), 2168-2195.

Chen, J. S., Hou, J. L., Wu, S. M., & Chang Chien, Y. W. (2009). Constructing investment strategy portfolios by combination genetic algorithms. *Expert Systems with Applications, 36*(2, Part 2), 3824-3828.

Chen, J. S., & Lin, Y. T. (2009). A partitioned portfolio insurance strategy by a relational genetic algorithm. *Expert Systems with Applications, 36*(2, Part 2), 2727-2734.

Chen, L., McPhee, J., & Yeh, W. W. G. (2007). A diversified multiobjective GA for optimizing reservoir rule curves. *Advances in Water Resources, 30*(5), 1082-1093.

Chen, Z., & Wang, Y. (2008). Two-sided coherent risk measures and their application in realistic portfolio optimization. *Journal of Banking & Finance, 32*(12), 2667-2673.

Daníelsson, J., Jorgensen, B., Vries, C., & Yang, X. (2008). Optimal portfolio allocation under the probabilistic VaR constraint and incentives for financial innovation. *Annals of Finance, 4*(3), 345-367.

Deb, K. (Ed.). (2001). *Multi-objective optimization using evolutionary algorithms* Chichester ; New York : John Wiley & Sons.

Dedieu, S., Pibouleau, L., Azzaro Pantel, C., & Domenech, S. (2003). Design and retrofit of multiobjective batch plants via a multicriteria genetic algorithm. *Computers & Chemical Engineering, 27*(12), 1723-1740.

Dietz, A., Azzaro Pantel, C., Pibouleau, L., & Domenech, S. (2008). Strategies for multiobjective genetic algorithm development: Application to optimal batch plant design in process systems engineering. *Computers & Industrial Engineering, 54*(3), 539-569.

Doran, J. S., Peterson, D. R., & Wright, C. (2010). Confidence, opinions of market efficiency, and investment behavior of finance professors. *Journal of Financial Markets, 13*(1), 174-195.

Fang, S. C., Rajasekera, J. R., & Tsao, H. S. J. (1997). *Entropy optimization and mathematical programming* Kluwer Academic Publisheres.

Fang, Y., Lai, K. K., & Wang, S. Y. (2006). Portfolio rebalancing model with transaction costs based on fuzzy decision theory. *European Journal of Operational Research, 175*(2), 879-893.

Frankfurter, G. M., & McGoun, E. G. (2002). Resistance is futile: the assimilation of behavioral finance. *Journal of Economic Behavior & Organization, 48*(4), 375-389.

Gatev, E., & Strahan, P. E. (2009). Liquidity risk and syndicate structure. *Journal of Financial Economics, 93*(3), 490-504.

Goel, T., Vaidyanathan, R., Haftka, R. T., Shyy, W., Queipo, N. V., & Tucker, K. (2007). Response surface approximation of Pareto optimal front in multi-objective optimization. *Computer Methods in Applied Mechanics and Engineering, 196*(4-6), 879-893.

Gutiérrez-Antonio, C., & Briones-Ramírez, A. (2009). Pareto front of ideal Petlyuk sequences using a multiobjective genetic algorithm with constraints. [doi: DOI: 10.1016/j.compchemeng.2008.11.004]. *Computers & Chemical Engineering, 33*(2), 454-464.

Hasuike, T., Katagiri, H., & Ishii, H. (2009). Portfolio selection problems with random fuzzy variable returns. *Fuzzy Sets and Systems, 160*(18), 2579-2596.

Huang, J., & Wang, J. (2010). Market liquidity, asset prices, and welfare. *Journal of Financial Economics, 95*(1), 107-127.

Huang, K. Y., & Jane, C. J. (2009). A hybrid model for stock market forecasting and portfolio selection based on ARX, grey system and RS theories. *Expert Systems with Applications, 36*(3, Part 1), 5387-5392.

Johnson, T. C. (2006). Dynamic liquidity in endowment economies. *Journal of Financial Economics, 80*(3), 531-562.

Kapur, J. N. (1990). *Maximum-entropy models in science and engineering* New Delhi: Wiley Eastern Limited.

Lin, C. C., & Liu, Y. T. (2008). Genetic algorithms for portfolio selection problems with minimum transaction lots. *European Journal of Operational Research, 185*(1), 393-404.

Lin, P. C., & Ko, P. C. (2009). Portfolio value-at-risk forecasting with GA-based extreme value theory. *Expert Systems with Applications, 36*(2, Part 1), 2503-2512.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance, 7*(1), 77-91.

Markowitz, H. M. (1959). *Portfolio selection: efficient diversification of investments* Wiley : New York.

Marseguerra, M., Zio, E., & Podofillini, L. (2004). A multiobjective genetic algorithm approach to the optimization of the technical specifications of a nuclear safety system. *Reliability Engineering & System Safety, 84*(1), 87-99.

Mitchell, M. (Ed.). (1998). *An Introduction to Genetic Algorithms*: MIT press.

Nawaz Ripon, K. S., Kwong, S., & Man, K. F. (2007). A real-coding jumping gene genetic algorithm (RJGGA) for multiobjective optimization. *Information Sciences, 177*(2), 632-654.

Osman, M. S., Abo Sinna, M. A., & Mousa, A. A. (2005a). An effective genetic algorithm approach to multiobjective resource allocation problems (MORAPs). *Applied Mathematics and Computation, 163*(2), 755-768.

Osman, M. S., Abo Sinna, M. A., & Mousa, A. A. (2005b). An effective genetic algorithm approach to multiobjective routing problems (MORPs). *Applied Mathematics and Computation, 163*(2), 769-781.

Pelsser, A., & Vorst, T. (1996). Transaction costs and efficiency of portfolio strategies. *European Journal of Operational Research, 91*(2), 250-263.

Rossi, S. P. S., Schwaiger, M. S., & Winkler, G. (2009). How loan portfolio diversification affects risk, efficiency and capitalization: A managerial behavior model for Austrian banks. *Journal of Banking & Finance, 33*(12), 2218-2226.

Shefrin, H. (2006). The Role of Behavioral Finance in Risk Management. In K. O. Michael (Ed.), *Risk Management* (pp. 653-676). Boston: Academic Press.

Soleimani, H., Golmakani, H. R., & Salimi, M. H. (2009). Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm. *Expert Systems with Applications, 36*(3, Part 1), 5058-5063.

Suga, K., Kato, S., & Hiyama, K. (2009). Structural analysis of Pareto-optimal solution sets for multi-objective optimization: An application to outer window design problems using Multiple Objective Genetic Algorithms. *Building and Environment, In Press, Corrected Proof*.

Wilding, T. (2003). Using genetic algorithms to construct portfolios. In S. Stephen & S. Alan (Eds.), *Advances in Portfolio Construction and Implementation* (pp. 135-160). Oxford: Butterworth-Heinemann.

Ⅶ Appendix Ⅰ

Ten firms with two objective (return and covariance)									
0.0010	0.1116	0.1127	0.1100	0.1118	0.1110	0.1112	0.1108	0.1102	0.1097
0.9910	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0013	0.0013	0.0013
0.6467	0.0173	0.0493	0.0309	0.0149	0.0971	0.0319	0.0183	0.0164	0.0773
0.8251	0.0064	0.0185	0.0109	0.0054	0.0629	0.0089	0.0066	0.0058	0.0498
0.9357	0.0016	0.0030	0.0021	0.0018	0.0321	0.0017	0.0016	0.0015	0.0197

0.7801	0.0108	0.0224	0.0147	0.0091	0.0753	0.0185	0.0108	0.0102	0.0484
0.2879	0.0816	0.0592	0.0446	0.1409	0.1215	0.0655	0.0474	0.0462	0.1053
0.7116	0.0116	0.0496	0.0268	0.0087	0.0729	0.0147	0.0125	0.0107	0.0810
0.8886	0.0037	0.0070	0.0046	0.0033	0.0506	0.0053	0.0036	0.0034	0.0305
0.9488	0.0016	0.0029	0.0021	0.0016	0.0272	0.0018	0.0017	0.0015	0.0113
0.2087	0.0877	0.0659	0.0430	0.1520	0.1525	0.0730	0.0481	0.0464	0.1226
0.9128	0.0019	0.0038	0.0026	0.0021	0.0450	0.0020	0.0018	0.0016	0.0272
0.6978	0.0150	0.0475	0.0262	0.0101	0.0786	0.0201	0.0158	0.0142	0.0746
0.8078	0.0050	0.0309	0.0170	0.0051	0.0578	0.0064	0.0056	0.0044	0.0601
0.1786	0.0833	0.0933	0.0825	0.0780	0.1260	0.0934	0.0831	0.0820	0.0998
0.9059	0.0021	0.0054	0.0035	0.0023	0.0459	0.0023	0.0021	0.0018	0.0293
0.9737	0.0012	0.0016	0.0014	0.0013	0.0107	0.0012	0.0014	0.0013	0.0070
0.7909	0.0048	0.0366	0.0221	0.0044	0.0578	0.0043	0.0048	0.0038	0.0713
0.4522	0.0338	0.0569	0.0453	0.0334	0.1563	0.0746	0.0339	0.0328	0.0808
0.6078	0.0227	0.0490	0.0309	0.0174	0.1115	0.0405	0.0231	0.0218	0.0755
0.8854	0.0032	0.0129	0.0078	0.0031	0.0450	0.0036	0.0033	0.0027	0.0329
0.8243	0.0042	0.0306	0.0186	0.0039	0.0481	0.0038	0.0043	0.0034	0.0595
0.6719	0.0216	0.0349	0.0218	0.0140	0.1007	0.0365	0.0221	0.0209	0.0558
0.5543	0.0322	0.0521	0.0327	0.0209	0.1214	0.0503	0.0323	0.0313	0.0727
0.2922	0.0740	0.0662	0.0469	0.1216	0.1315	0.0650	0.0457	0.0442	0.1128
0.4059	0.0653	0.0495	0.0324	0.1128	0.1166	0.0544	0.0361	0.0348	0.0925
0.4753	0.0399	0.0605	0.0356	0.0224	0.1456	0.0619	0.0402	0.0390	0.0798
0.6248	0.0236	0.0411	0.0298	0.0206	0.1076	0.0455	0.0238	0.0229	0.0605
0.4993	0.0331	0.0587	0.0374	0.0230	0.1415	0.0592	0.0333	0.0321	0.0825
0.9910	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0013	0.0013	0.0013
0.7467	0.0067	0.0486	0.0252	0.0056	0.0637	0.0071	0.0078	0.0058	0.0829
0.3078	0.0735	0.0614	0.0395	0.1263	0.1360	0.0617	0.0411	0.0395	0.1132
0.9860	0.0011	0.0012	0.0012	0.0011	0.0041	0.0011	0.0013	0.0013	0.0025
0.8395	0.0038	0.0269	0.0164	0.0036	0.0466	0.0035	0.0039	0.0031	0.0535
0.3165	0.0630	0.0763	0.0552	0.0462	0.1486	0.0819	0.0630	0.0619	0.0877
0.3182	0.0724	0.0605	0.0389	0.1243	0.1342	0.0607	0.0405	0.0389	0.1115
0.7621	0.0063	0.0439	0.0228	0.0053	0.0624	0.0066	0.0073	0.0054	0.0778
0.7070	0.0135	0.0487	0.0260	0.0181	0.0698	0.0126	0.0111	0.0092	0.0840
0.4345	0.0410	0.0640	0.0396	0.0256	0.1585	0.0706	0.0419	0.0399	0.0845
0.9602	0.0015	0.0024	0.0018	0.0014	0.0201	0.0016	0.0016	0.0014	0.0086
0.3549	0.0551	0.0740	0.0547	0.0441	0.1419	0.0761	0.0556	0.0540	0.0898
0.6574	0.0205	0.0383	0.0246	0.0152	0.1043	0.0363	0.0207	0.0197	0.0633

0.8647	0.0041	0.0141	0.0083	0.0038	0.0527	0.0056	0.0042	0.0037	0.0392
0.2526	0.0819	0.0648	0.0451	0.1398	0.1383	0.0686	0.0473	0.0457	0.1160
0.9263	0.0018	0.0035	0.0024	0.0019	0.0383	0.0019	0.0018	0.0016	0.0212
0.5235	0.0287	0.0533	0.0387	0.0261	0.1358	0.0590	0.0289	0.0277	0.0784
0.9692	0.0013	0.0022	0.0017	0.0014	0.0125	0.0014	0.0015	0.0014	0.0084
0.5865	0.0253	0.0508	0.0318	0.0184	0.1168	0.0440	0.0257	0.0244	0.0763
0.7330	0.0105	0.0428	0.0244	0.0096	0.0716	0.0161	0.0110	0.0095	0.0715
0.8080	0.0049	0.0309	0.0163	0.0043	0.0589	0.0051	0.0055	0.0042	0.0624
0.8719	0.0035	0.0151	0.0091	0.0035	0.0487	0.0041	0.0037	0.0030	0.0375
0.0010	0.1116	0.1127	0.1100	0.1118	0.1110	0.1112	0.1108	0.1102	0.1097
0.0569	0.1052	0.1001	0.0920	0.1226	0.1222	0.1009	0.0939	0.0930	0.1132

Ten firms with three objective (return, covariance and skewness)									
0.9910	0.0010	0.0010	0.0010	0.0020	0.0010	0.0010	0.0010	0.0010	0.0010
0.1445	0.2203	0.1007	0.0643	0.0953	0.0198	0.0640	0.0955	0.0951	0.1005
0.4260	0.0351	0.1344	0.0369	0.0346	0.0822	0.0345	0.0378	0.0146	0.1634
0.1502	0.0682	0.1278	0.0709	0.0682	0.0993	0.1183	0.0697	0.1467	0.0801
0.4572	0.0315	0.1263	0.0342	0.0313	0.0867	0.0613	0.0344	0.0230	0.1141
0.9910	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
0.8126	0.0086	0.0602	0.0093	0.0085	0.0328	0.0083	0.0096	0.0056	0.0450
0.2756	0.0506	0.1235	0.0556	0.0506	0.1385	0.0804	0.0550	0.0972	0.0728
0.3111	0.0510	0.1231	0.0534	0.0508	0.0863	0.0777	0.0532	0.0858	0.1071
0.2659	0.1235	0.0997	0.0710	0.0548	0.0592	0.0661	0.0856	0.0813	0.0931
0.6160	0.0244	0.0898	0.0248	0.0235	0.0546	0.0232	0.0255	0.0108	0.1075
0.2142	0.1494	0.1101	0.0598	0.0767	0.0476	0.0698	0.0778	0.0912	0.1033
0.6238	0.0225	0.0913	0.0242	0.0223	0.0572	0.0232	0.0242	0.0122	0.0992
0.7669	0.0120	0.0677	0.0130	0.0119	0.0381	0.0117	0.0133	0.0070	0.0589
0.6904	0.0170	0.0839	0.0183	0.0169	0.0486	0.0169	0.0187	0.0095	0.0799
0.4379	0.0344	0.1288	0.0365	0.0339	0.0875	0.0351	0.0373	0.0184	0.1498
0.7047	0.0162	0.0814	0.0175	0.0161	0.0470	0.0163	0.0177	0.0101	0.0734
0.7781	0.0112	0.0684	0.0124	0.0113	0.0380	0.0112	0.0125	0.0086	0.0488
0.5657	0.0248	0.1140	0.0267	0.0244	0.0719	0.0248	0.0270	0.0155	0.1054
0.9045	0.0062	0.0204	0.0071	0.0064	0.0127	0.0107	0.0072	0.0051	0.0206
0.2246	0.1900	0.0937	0.0611	0.0848	0.0218	0.0588	0.0875	0.0861	0.0917
0.5351	0.0262	0.1234	0.0285	0.0259	0.0757	0.0262	0.0287	0.0156	0.1148
0.9845	0.0014	0.0027	0.0014	0.0014	0.0021	0.0014	0.0014	0.0014	0.0024

0.5982	0.0245	0.0955	0.0261	0.0241	0.0630	0.0316	0.0265	0.0163	0.0943
0.8585	0.0081	0.0309	0.0087	0.0086	0.0264	0.0082	0.0089	0.0083	0.0341
0.8342	0.0080	0.0503	0.0086	0.0080	0.0292	0.0078	0.0089	0.0059	0.0396
0.1968	0.0687	0.1129	0.0861	0.0684	0.0792	0.0959	0.0871	0.1213	0.0834
0.6084	0.0222	0.0983	0.0240	0.0221	0.0627	0.0334	0.0243	0.0147	0.0899
0.9105	0.0043	0.0273	0.0046	0.0043	0.0152	0.0060	0.0048	0.0039	0.0194
0.3814	0.0372	0.1233	0.0430	0.0370	0.1344	0.1117	0.0431	0.0332	0.0559
0.7948	0.0107	0.0596	0.0113	0.0105	0.0336	0.0103	0.0116	0.0061	0.0519
0.8380	0.0077	0.0514	0.0082	0.0075	0.0282	0.0074	0.0084	0.0052	0.0385
0.8827	0.0069	0.0270	0.0074	0.0068	0.0168	0.0135	0.0074	0.0047	0.0275
0.3164	0.1051	0.1119	0.0490	0.0575	0.0828	0.0854	0.0609	0.0549	0.0762
0.4963	0.0313	0.1141	0.0332	0.0306	0.0801	0.0358	0.0339	0.0182	0.1263
0.3217	0.0501	0.1217	0.0525	0.0499	0.0852	0.0762	0.0523	0.0839	0.1063
0.5529	0.0264	0.1081	0.0283	0.0261	0.0688	0.0336	0.0288	0.0144	0.1125
0.8722	0.0083	0.0286	0.0082	0.0077	0.0184	0.0144	0.0084	0.0061	0.0284
0.6706	0.0184	0.0873	0.0195	0.0182	0.0508	0.0181	0.0200	0.0092	0.0880
0.8571	0.0076	0.0410	0.0081	0.0075	0.0239	0.0078	0.0083	0.0059	0.0335
0.7180	0.0161	0.0771	0.0169	0.0150	0.0447	0.0161	0.0173	0.0111	0.0679
0.1445	0.2203	0.1007	0.0643	0.0953	0.0198	0.0640	0.0955	0.0951	0.1005
0.5885	0.0242	0.0976	0.0262	0.0239	0.0716	0.0258	0.0267	0.0171	0.0984
0.9710	0.0021	0.0055	0.0020	0.0022	0.0056	0.0026	0.0021	0.0027	0.0044
0.6379	0.0209	0.0904	0.0228	0.0208	0.0654	0.0238	0.0230	0.0166	0.0786
0.8898	0.0065	0.0253	0.0070	0.0065	0.0158	0.0127	0.0070	0.0045	0.0257
0.9282	0.0050	0.0173	0.0057	0.0052	0.0100	0.0047	0.0059	0.0049	0.0139
0.3856	0.0380	0.1233	0.0431	0.0378	0.1259	0.0923	0.0431	0.0425	0.0683
0.6794	0.0177	0.0855	0.0188	0.0176	0.0497	0.0173	0.0193	0.0087	0.0861
0.9910	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0020	0.0010
0.6453	0.0220	0.0855	0.0225	0.0212	0.0514	0.0210	0.0232	0.0101	0.0980
0.2756	0.0506	0.1235	0.0556	0.0506	0.1385	0.0804	0.0550	0.0972	0.0728
0.1502	0.0682	0.1278	0.0709	0.0682	0.0993	0.1183	0.0697	0.1467	0.0801
Ten firms with three objective (return, covariance and entropy)									
0.9909	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
0.9909	0.0010	0.0010	0.0010	0.0010	0.0010	0.0020	0.0010	0.0010	0.0010
0.0775	0.1783	0.0653	0.0654	0.1791	0.0622	0.0649	0.0679	0.1736	0.0649
0.9179	0.0067	0.0083	0.0069	0.0099	0.0110	0.0106	0.0079	0.0088	0.0122
0.6825	0.0318	0.0347	0.0274	0.0370	0.0385	0.0397	0.0313	0.0322	0.0452
0.6111	0.0380	0.0416	0.0356	0.0451	0.0480	0.0478	0.0381	0.0420	0.0528

0.4611	0.0585	0.0569	0.0536	0.0619	0.0631	0.0609	0.0557	0.0625	0.0662
0.0999	0.0987	0.0997	0.1014	0.1000	0.0999	0.0998	0.1019	0.1001	0.0995
0.8610	0.0128	0.0137	0.0132	0.0167	0.0186	0.0175	0.0128	0.0163	0.0180
0.5148	0.0535	0.0506	0.0478	0.0568	0.0564	0.0545	0.0496	0.0573	0.0591
0.8759	0.0071	0.0106	0.0109	0.0157	0.0208	0.0178	0.0086	0.0150	0.0178
0.9531	0.0035	0.0042	0.0040	0.0061	0.0070	0.0066	0.0037	0.0057	0.0064
0.9676	0.0023	0.0029	0.0029	0.0041	0.0051	0.0044	0.0026	0.0037	0.0046
0.5732	0.0448	0.0464	0.0417	0.0487	0.0496	0.0502	0.0442	0.0474	0.0541
0.6984	0.0202	0.0284	0.0276	0.0353	0.0473	0.0410	0.0246	0.0379	0.0393
0.6712	0.0295	0.0335	0.0296	0.0391	0.0438	0.0417	0.0303	0.0361	0.0458
0.8759	0.0117	0.0123	0.0117	0.0151	0.0163	0.0153	0.0114	0.0147	0.0160
0.6183	0.0365	0.0404	0.0349	0.0443	0.0480	0.0473	0.0370	0.0417	0.0516
0.1872	0.0889	0.0888	0.0905	0.0885	0.0895	0.0892	0.0902	0.0984	0.0897
0.7134	0.0192	0.0260	0.0264	0.0354	0.0450	0.0388	0.0223	0.0342	0.0394
0.8880	0.0102	0.0112	0.0110	0.0134	0.0148	0.0140	0.0105	0.0133	0.0141
0.2128	0.1251	0.0680	0.0684	0.1272	0.0679	0.0689	0.0696	0.1242	0.0688
0.9099	0.0068	0.0087	0.0076	0.0110	0.0129	0.0120	0.0080	0.0100	0.0132
0.9297	0.0080	0.0067	0.0062	0.0093	0.0080	0.0084	0.0064	0.0085	0.0086
0.6028	0.0416	0.0416	0.0372	0.0470	0.0475	0.0468	0.0393	0.0453	0.0511
0.8438	0.0121	0.0147	0.0127	0.0198	0.0223	0.0209	0.0133	0.0169	0.0239
0.9370	0.0072	0.0068	0.0066	0.0071	0.0070	0.0071	0.0066	0.0076	0.0070
0.8502	0.0114	0.0139	0.0128	0.0194	0.0224	0.0198	0.0118	0.0175	0.0214
0.3220	0.0740	0.0743	0.0732	0.0741	0.0751	0.0753	0.0733	0.0835	0.0759
0.6860	0.0222	0.0299	0.0290	0.0367	0.0481	0.0421	0.0263	0.0392	0.0407
0.3305	0.0732	0.0733	0.0722	0.0733	0.0742	0.0743	0.0723	0.0826	0.0749
0.3794	0.0682	0.0686	0.0695	0.0691	0.0688	0.0689	0.0700	0.0695	0.0687
0.2016	0.0867	0.0877	0.0888	0.0878	0.0892	0.0888	0.0890	0.0928	0.0885
0.2860	0.0779	0.0789	0.0801	0.0795	0.0796	0.0794	0.0804	0.0797	0.0792
0.8035	0.0110	0.0168	0.0171	0.0248	0.0331	0.0279	0.0136	0.0238	0.0282
0.7828	0.0149	0.0200	0.0198	0.0267	0.0335	0.0292	0.0174	0.0255	0.0301
0.7688	0.0200	0.0222	0.0197	0.0289	0.0325	0.0301	0.0195	0.0261	0.0329
0.4740	0.0570	0.0568	0.0561	0.0592	0.0595	0.0590	0.0564	0.0626	0.0603
0.8957	0.0086	0.0099	0.0086	0.0132	0.0144	0.0141	0.0091	0.0113	0.0157
0.4897	0.0409	0.0573	0.0504	0.0461	0.0712	0.0654	0.0544	0.0754	0.0497
0.3969	0.0648	0.0656	0.0640	0.0668	0.0679	0.0678	0.0647	0.0726	0.0697